9/29/17 "By failing to prepare, you are preparing to fail." -Ben Franklin

HW: Have a Great Weekend!

AIM: What is the Limit Definition of Derivative?

Warm Up:

1) Evaluate and simplify the difference quotient for

$$f(x) = 3x^2 - 2x + 5$$

$$\frac{3(x+h)^2 - 2(x+h) + 5 - 3x^2 + 2x - 5}{h}$$

$$\frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-2x} - 2h \cancel{+5} \cancel{-3x^2} \cancel{+2x} \cancel{-5}}{h}$$

$$\frac{6xh + 3h^2 - 2h}{h} = \boxed{6x + 3h - 2}$$

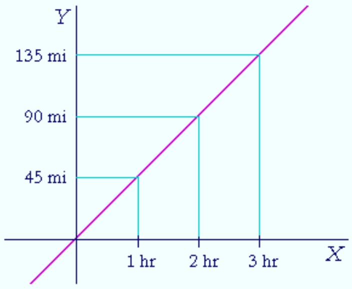
Differentiation: the process of looking at the way a function changes from one point to another.

Slope: The rate of change of a function

(Derivative)

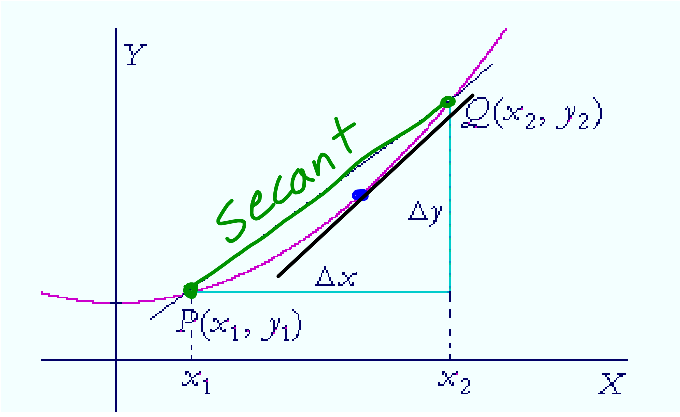
A straight line has one and only one slope.

If x represents time, for example, and y represents distance, then a



straight line graph that relates them indicates constant speed. 45 miles per hour, say -- at every moment of time.

What about the slope (rate of change) of a curve?

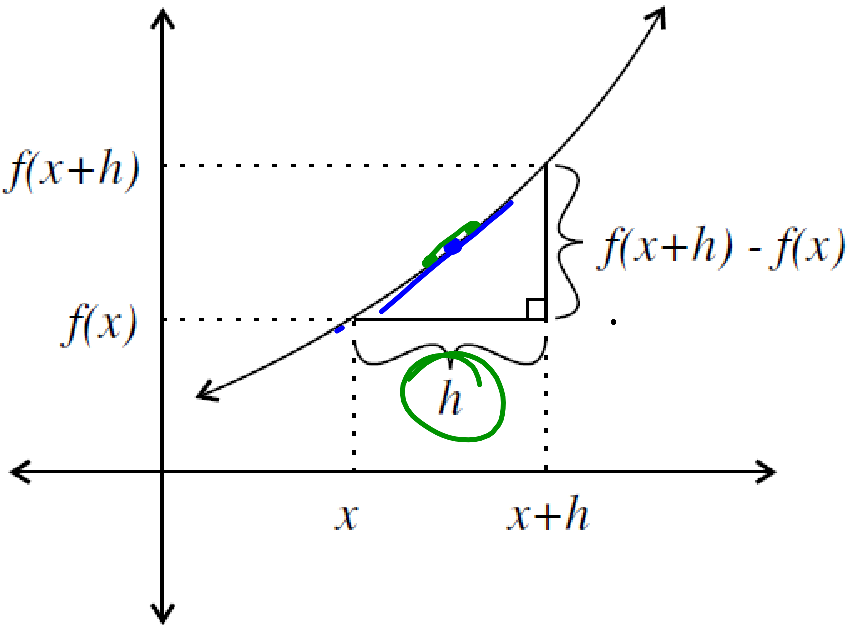


The slope of a secant line will give us Average Rate Of Change of a function between 2 points.

A.R.O.C.

The slope of a tangent line at a point on the function gives us the Instantaneous Rate of Change of the function at that point.

I.R.O.C.



The smaller the "h" gets, the closer we are to finding the slope at the point (x, $f(x)$).

Derivative: (Slope)

$$\frac{dy}{dx} = y' = f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Notice that $f'(x)$ is the derivative only if the limit exists. If the limit does not exist at particular x-values then we say that the function is not differentiable at those x-values.

Example 4 : Find the derivative of $f(x) = x^2 + 3$.

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2+3 - (x^2+3)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+3} \cancel{-x^2} \cancel{-3}}{h}$$

$$= \lim_{h\to 0} \frac{2x\cancel{h} + h^{\cancel{2}}}{\cancel{h}}$$

$$= \lim_{h\to 0} 2x + h$$

$$\boxed{f'(x) = 2x}$$

Exercises:

1. Using the method outlined above, find $f'(x)$ for each of the following functions. That is, use

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   (a) $f(x) = x^2 + 2$
   (b) $f(x) = 3x - 5$
   (c) $f(x) = 3 - x^2$
   (d) $f(x) = 4x + 5$
   (e) $f(x) = 2 - x$

Find the derivative of

$$g(x) = x^3 + 2x^2 + 4$$

$$g'(x) = \lim_{h \to 0} \frac{(x+h)^3 + 2(x+h)^2 + 4 - (x^3 + 2x^2 + 4)}{h}$$

$$\frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{2x^2} + 4xh + 2h^2 \cancel{+4} \cancel{-x^3} \cancel{-2x^2} \cancel{-4}}{h}$$

$$\frac{3x^2\cancel{h} + 3xh^{\cancel{2}} + h^{\cancel{3}\,2} + 4x\cancel{h} + 2h^{\cancel{2}}}{\cancel{h}}$$

$$g'(x) = \lim_{h \to 0} 3x^2 + 3xh + h^2 + 4x + 2h$$

$$\boxed{g'(x) = 3x^2 + 4x}$$

$$g(x) = x^3 + 2x^2 + 4$$

HW: Limit definition of derivative
#1-5

Test 2 on 10/16